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Highly charged ions, quantum-electrodynamics, and the electron mass

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Abstract

High precision experiments on the magnetic moment of hydrogen-like ions confined in a Penning trap have provided the most stringent test of bound-state quantum-electrodynamic calculations. Experiments have been performed on single C^{5+} and O^{7+} ions. These experiments are briefly reviewed and prospects for future improvements and extension to other systems are discussed.

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Keywords: Quantum-electrodynamics; Electron mass; Magnetic moment

1. Introduction

The physics of highly charged ions has attracted increasing interest in recent years. The additional charges as compared to neutral or singly ionized atoms enhanced in many cases effects that are barely detectable otherwise, lead to increased precision in many experiments, and allow for novel applications. To mention a few recent achievements, highly charged ions have been used as probes in surface studies [1], they have led to increased accuracy in mass spectrometry [2], allowed for the observation of X-ray transitions of astrophysical interest [3], improved the precision of fundamental constants [4], and found applications in medical therapy [5]. Few-electrons ions such as hydrogen- or helium-like systems are of particular interest, since they can be treated theoretically with high precision, and the comparison of experimental and theoretical results provides a sensitive test of our understanding of the atomic structure. On the experimental side a number of sources have become available, which are capable of delivering almost any desired ion species up to bare uranium. Ion storage rings have been in operation for many years. They create highly charged ions by acceleration of low charged species and electron stripping in thin foils, which can either be used for experiments while circulating in the storage ring or can be extracted and made available for other purposes. Improved beam handling and cooling techniques have led to stored ion beams of low velocity spread and high brilliance. Ref. [6] reviews the present status of these devices. Alternatively electron-beam ion sources (EBIS) or traps (EBIT) create ions by successive ionization of low charged ions stored in the radial space charge potential of an electron beam. Again investigation is possible while the ions remain inside the EBIT or after extraction outside the device. Ref. [7] gives typical examples.

The theoretical treatment of highly charged ions bears some difficulties compared to neutral atoms because the high nuclear charge makes relativistic effects particularly large and standard approximation methods give less reliable results. Also quantum electrodynamic (QED) effects become more important. The standard method to calculate QED contributions is to calculate Feynman diagrams in a perturbative way according to the number of exchange photons or virtual electron–positron pairs. The expansion parameter is the fine structure constant $\alpha \approx 1/137$. In case of highly charged ions this becomes much more difficult since the electron can no longer be described by a plane wave as in the free particle case but its wave function is the solution of the Dirac equation. Moreover, the expansion parameter in the perturbation expansion for the coupling to the nuclear coulomb field (i.e., the binding) is now $(Z\alpha)$, Z being the nuclear charge. Consequently higher orders of the perturbation series become

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more significant than in the case of a free particle and it seems virtually impossible to perform the calculation to a similar level of accuracy as it has been obtained for the free electron. In spite of these difficulties substantial progress has been made in recent years for simple highly charged ions having hydrogen- or lithium-like electronic configurations [8,9].

In this contribution, we will describe recent experiments, which aim at precise measurements of the magnetic moment of the electron bound in hydrogen-like ions. The comparison of the results to theoretical expectations represents a stringent test of bound-state QED. It turns out that the technique developed for these experiments may serve to determine other quantities of fundamental interest such as magnetic moments of bare nuclei free of diamagnetic shielding, high precision mass spectrometry, or hyperfine spectroscopy on hydrogenic systems.

2. The magnetic moment of the bound electron

The magnetic moment $\vec{\mu}$ associated with the spin $\vec{s}$ of the electron is usually expressed by the g factor defined by

$$g = \frac{m}{e}\frac{\mu}{s} \quad (1)$$

where m and e are the electron's mass and charge, respectively. It is a dimensionless number and the solution of the Dirac equation for a free particle gives the value $g = 2$. The exchange of virtual photons and virtual pair production as considered by the theory of quantum electrodynamics (QED) changes this value by about 1 part in thousand. The presently best theoretical value as quoted from the CODATA compilation of fundamental constants is

$$g = 2(1 + a); \quad a = 0.0011596521852(38) \quad (2)$$

In a series of experiments Dehmelt and coworkers [10] have determined experimentally the free electron's g factor and obtained

$$a = 0.001159652188(4) \quad (3)$$

The agreement between theory and experiment represents the best low energy test of QED for free particles.

For the bound electrons in hydrogen-like systems of nuclear charge Z some additional differences to the free particles value for g occur. The main part comes from the solution of the Dirac equation. It as been analytically obtained by Breit [11]:

$$g = \frac{2}{3}(1 + 2\sqrt{1 - Z^2\alpha^2}) \quad (4)$$

Bound-state quantum electrodynamic (BS-QED) contributions have been calculated for several highly charged ions [8,12–16]. They are about 3 orders of magnitude smaller than the Breit correction. Not yet calculated higher order terms determine the uncertainty of the theoretical g factor for low-Z ions to about 1 part in 10^{10}. For higher values of Z nuclear volume and nuclear recoil contributions become significant. These are difficult to calculate accurately and will ultimately limit the comparison of experimental and theoretical g factors as BS-QED test. Fig. 1 summarizes the different contributions to the g factor as a function of the nuclear charge. From Fig. 1 it becomes evident that a test of the BS-QED calculations requires for low values of Z a precision of about a part in 10^9 in order to be significant, while for high-Z ions a precision of a part in 10^7 will be sufficient.

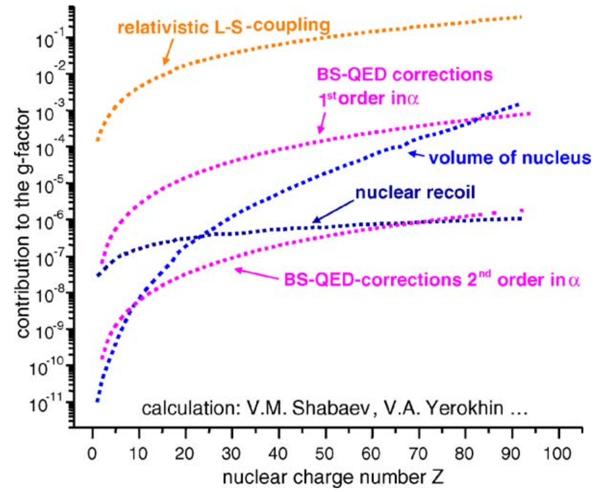

Fig. 1. Contributions to the g factor in hydrogen-like ions of different nuclear charges Z.

3. Experimental determination of the g factor

We have performed experiments on hydrogen-like low-Z ions carbon C^{5+} and oxygen O^{7+} for tests of BS-QED calculations. The experiments are part of a collaboration between the heavy ion research facility GSI and the physics department at the University of Mainz. Detailed descriptions of the experiments can be found in Refs. [17,18]. In short a single C^{5+} or O^{7+} ion is confined in an open-endcap cylindrical Penning trap. The trap consists in a stack of five cylindrical electrodes, a central ring, two endcaps and in between two correction electrodes which serve to make the electric trap potential near the trap center as harmonic as possible by application of proper correction voltages. A strong homogeneous magnetic field is directed along the trap axis. We call this trap "precision trap". A second trap of identical geometry ("analysis trap") is placed at 2.7 cm distance along the axis from the first one (Fig. 2). Its ring electrode is made of nickel, which distorts the magnetic field in a bottle-like manner. A single stored ion is confined in the trap and detected by the induced noise in LC-circuits attached to the trap electrodes and tuned to the ion oscillation frequencies in axial and radial directions, respectively. The ion is cooled by resistive cooling in all degrees of freedom by thermal contact to external resonance circuits. These circuits are maintained at liquid helium temperature and after a short time, determined by the quality-factor of the circuits, the ion assumes a similar temperature of about 4 K. The ion temperature is slightly above the environment temperature since noise from the attached electronics heats up the ions to a certain degree. The determination of the electron's g factor requires the measurement of the energy difference $\Delta E = g\mu_B B$ between the two spin directions in a known magnetic field B. $\mu_B = (e/m)\hbar$ is the Bohr magnetron. B can be calibrated by the cyclotron frequency $\omega_c = (q/M)B$ of the stored ion. Then g is

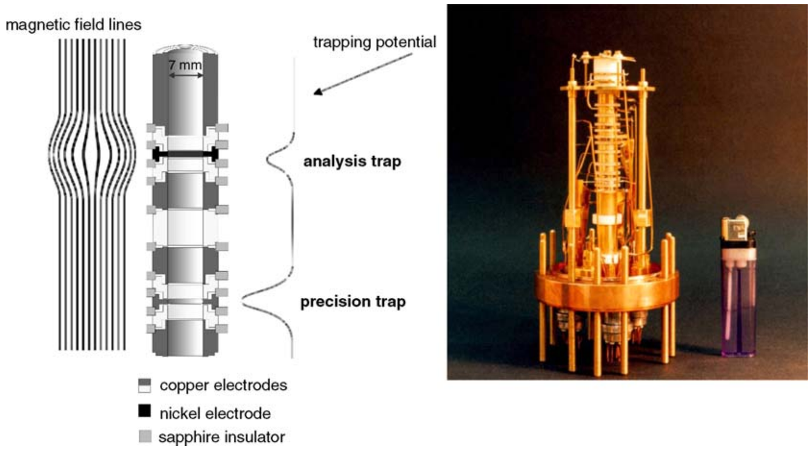

Fig. 2. Double trap arrangement for g factor determination. Two geometrically identical traps each consisting of a stack of five cylindrical electrodes of 7 mm inner diameter create two potential minima for ion storage. In the "analysis trap", the magnetic field is distorted in a bottle-like manner by a nickel ring electrode.

given by

$$g = 2\frac{\omega_L}{\omega_c}\frac{m}{M} \tag{5}$$

where q is the charge state of the ion of mass M and $\omega_L = \Delta E/\hbar$ the spin precession frequency. Thus, the determination of g requires a measurement of the frequencies ω_L and ω_c while the mass ratio m/M is taken from literature. The measurement of the ions oscillation frequencies is performed by a Fourier analysis of the noise induced by image charges of the oscillating ion in the trap electrodes. Figs. 3 and 4 show the results for the radial and axial directions, respectively. The maximum of the electronic signal appears at the "perturbed cyclotron frequency":

$$\omega_c' = \frac{\omega_c}{2} + \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}} \tag{6}$$

where $\omega_z = \sqrt{(qV)/(Mz_0^2)}$ is the axial oscillation frequency of the ion in a Penning trap with an endcap distance of $2z_0$. ω_c is obtained by the quadrature sum of the eigenfrequencies:

$$\omega_c^2 = \omega_c'^2 + \omega_z^2 + \omega_m^2 \tag{7}$$

Here $\omega_c' = \omega_c/2 - \sqrt{\omega_c^2/4 - \omega_z^2/2}$ is the magnetron frequency, a slow drift motion of the ion around the trap center. It is determined by sideband excitation to ω_z. The precision of ω_c is mainly determined by ω_c'. From Fig. 3 it is seen that this can be measured to the 10^{-10} level of accuracy. We note that the resonances appear as a maximum in the noise spectrum in radial direction but as a minimum in axial direction. This is because the ions were excited to about 1 eV energy in the radial direction to increase the signal strength. The minimum in the axial noise distribution can be understood when we consider that the total noise at the ions axial oscillation frequency is the sum of the thermally fluctuating noise in the electronic circuit and the induced noise from the ion. The induced voltage, however, has a phase difference of 180° with respect to the thermal noise. Consequently in the sum of both components the amplitude is reduced compared to the case when no ion is in the trap. The Zeeman splitting of the ground level of the hydrogen-like ion is measured by microwave induced spin flips. A successful spin flip is detected by means of the "continuous Stern-Gerlach ef-

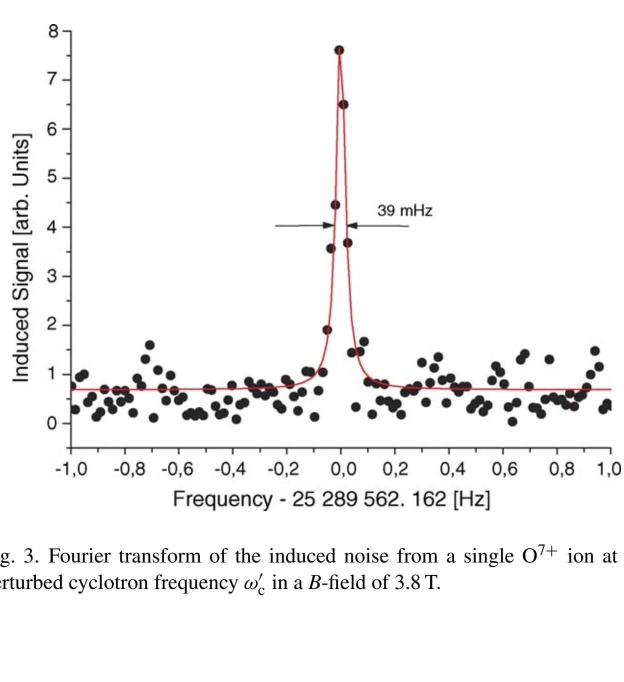

Fig. 3. Fourier transform of the induced noise from a single O^{7+} ion at the perturbed cyclotron frequency ω_c' in a B-field of 3.8 T.

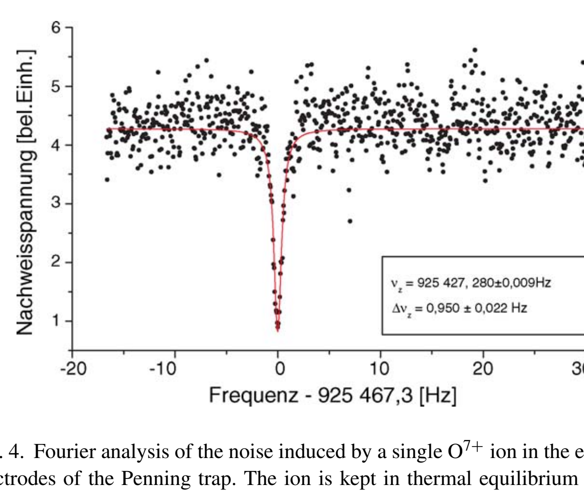

Fig. 4. Fourier analysis of the noise induced by a single O^{7+} ion in the end cap electrodes of the Penning trap. The ion is kept in thermal equilibrium with a resonant circuit across the endcaps.

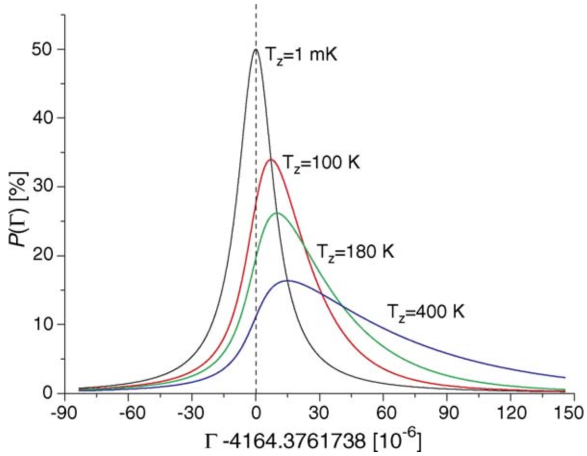

Fig. 5. Simulation of g-factor resonances for different axial temperatures of a single ion.

fect": when the ion is in the analysis trap the force acting on the spin magnetic moment in the inhomogeneous B-field adds to the electric trapping force acting on the charge and thus changes its axial oscillation frequency. The sign of the Stern-Gerlach force, however, depends on the direction of the spin with respect to the magnetic field. Thus, a spin flip manifests itself as a difference in the axial oscillation frequency. Its size depends on the field inhomogeneity. For the given geometry in our trap it amounts to 0.48 Hz at a total axial frequency of 369 kHz for O^{7+}. The measurement of such a small frequency difference represents a challenge and requires extremely stable trapping voltage since a change in the trapping potential of about 10 μV would mimic a spin flip. The narrow axial resonance shown in Fig. 4, however, demonstrates that frequency differences well below 0.5 Hz can be detected. Using this detection a maximum of induced spin flips per unit time occurs at a certain microwave frequency. The line shape of this resonance in the inhomogeneous B-field, however, is rather asymmetric due to the thermal fluctuations of the ions radial energy. Therefore, the ion is transported from the analysis trap, where the spin direction has been determined by a measurement of the axial frequency, into the precision trap, where spin flips are induced. Then it is transported back into the analysis trap and it is tested whether the spin has changed its direction or not. Thanks to the homogeneous field in the precision trap, the line shape is now symmetric apart from a small asymmetry due to residual inhomogeneities (Fig. 5). Together with the simultaneously determined cyclotron frequency in the precision trap a value for the g factor is obtained. The experimental values for O^{7+} and C^{5+} are given in Table 1 and compared to the most recent theoretical values. The error bars given in Table 1 are for the theoretical part given by estimates of non calculated higher order terms. For the experiments the first error is the sum of the statistical and systematical uncertainty, the second is due to the uncertainty of the electron mass. Within the limits of errors given experiment and theory agree.

Table 1
Experimental and theoretical g factors for C^{5+} and O^{7+}

	C^{5+}	O^{7+}
Experiment	2.0010415964(7)(44) [19]	2.0010470254(15)(44) [20]
Theory	2.00104159018(3) [8]	2.00004702032(11) [8]

4. The electron mass

From Table 1 it is evident that the uncertainty of the electron mass represents the largest contribution of the errors. Therefore, we can change our point of view and determine a value for the electron mass from a comparison of experimental and theoretical values:

$$m = \frac{g_{\text{theor}}}{2}\frac{\omega_c}{\omega_L}M \tag{8}$$

Taken the masses M of the carbon and oxygen ion, respectively, from the CODATA tables and using the most recent values for the theoretical g factors as quoted in Table 1 we obtain for the electron mass [8]

$$C^{5+}: m_e = 0.00054857990932(29) \tag{9}$$

$$O^{7+}: m_e = 0.00054857990960(41) \tag{10}$$

The 2002 Codata compilation of fundamental constants [21] lists as new value for the electron mass based on the g factor measurements:

$$m_e = 0.00054857990945(24) \tag{11}$$

Compared to the value listed in the 1998 CODATA tables [22] of $m_e = 0.0005485799110(12)$ this represents an improvement of a factor 5.

There are possibilities for improvement of this value in the near future. The main contributions to the errors in the present experiment have two sources: the measurement of the cyclotron frequency requires the excitation of the ion to a radial energy of about 1 eV. Due to residual inhomogeneities in the B-field this leads to a small energy dependence of the resonances, which has to be taken care of by measurements at different excitation amplitudes and extrapolation to zero energy. Recently, we showed that radial excitation can be avoided by coupling of axial and radial motions [23]. Application of this method to future measurements will reduce the error bar by about a factor of 3.

The linewidth of our g-factor resonance is mainly determined by the elevated axial temperature of the ion due to excitation from so far unidentified noise sources in the electronic devices attached to the trap. Simulations [24] have shown that lower temperatures will lead to significant reduction of the linewidth and of the residual line asymmetry (Fig. 5). It remains to be seen to which extent the electronic noise and consequently the ions axial temperature can be reduced.

5. Ions of higher nuclear charge

As seen from Fig. 1 the different contributions to the g factor of the electron in a hydrogen-like ion increase with increasing nuclear charge Z, approximately proportional to Z^2. Test of the BS-QED part will become more significant as long as other contributions are small or can be calculated reliably. We are preparing experiments at medium-high Z values where the nuclear contributions to g are reasonably small. We have constructed a new trap suited for experiments on H-like Ca^{19+} (Figs. 6 and 7). One significant difference to the previous trap is the addition of an EBIT-like trap, which allows for charge breeding of hydrogen like calcium ions. A computer code based on the work by Becker et al. [24] has been optimized and used to simulate the in-trap charge breeding process by electron-impact ionization (Fig. 8). Another problem arises from the fact that

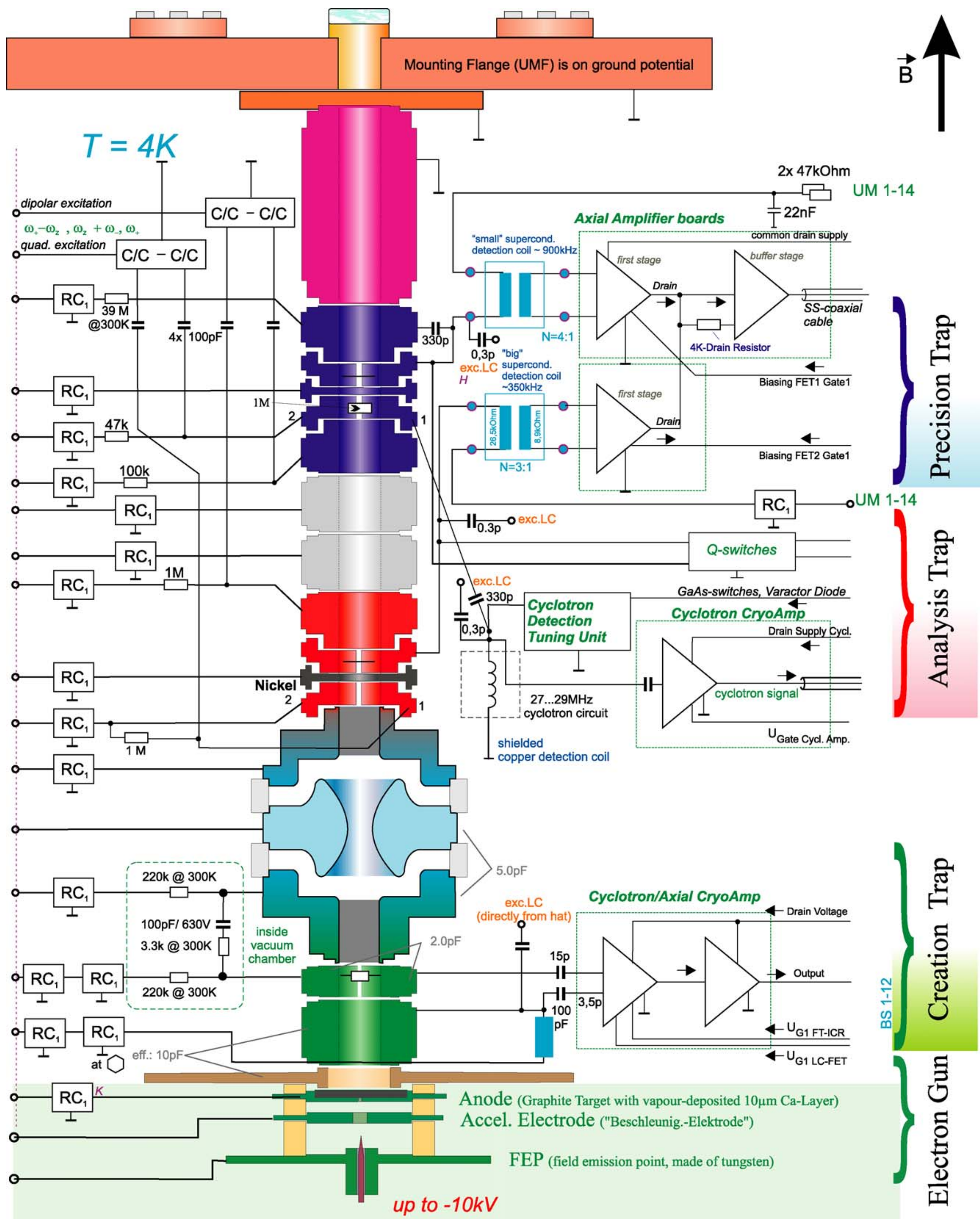

Fig. 6. Design of the triple trap for g factor measurements on Ca^{19+} including electronics for ion production, cooling and detection.

Fig. 7. Photograph of the assembled triple trap.

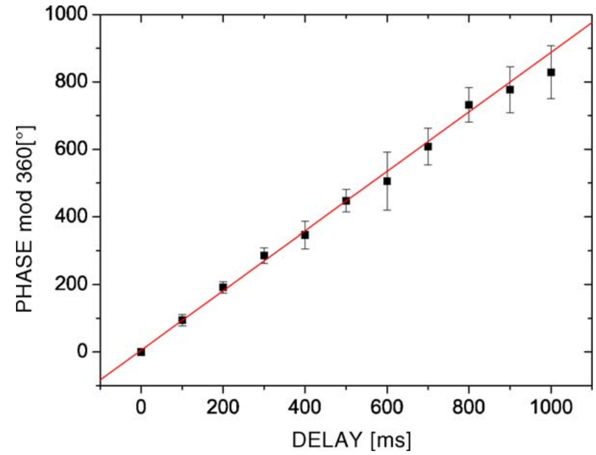

Fig. 9. Measured phase difference in the axial motion for a simulated spin flip of a stored ion as function of the measuring time. Phase differences of 45° can be resolved which corresponds to frequency a difference of 180 mHz.

the axial frequency difference upon a spin flip decreases with the mass and charge of the H-like ion as $(qM)^{-1/2}$. Under the conditions of the experiments on C^{5+} and O^{7+} it would amount to 0.18 Hz for calcium Ca^{19+} which is below our detection sensitivity. The new trap increases this value by a different geometry of the ferromagnetic ring electrodes which will produce a stronger *B*-field inhomogeneity. Moreover the detection of frequency differences in the axial motion will be replaced by a measurement of the phase difference in the axial oscillation, which appears when the spin direction changes sign. Tests have demonstrated that a phase difference corresponding to a frequency change of 0.09 Hz can be detected without ambiguity (Fig. 9) and further improvements are at hand [25]. Going to even higher nuclear charges requires loading of the trap from an outside source since the necessary electron energy for the high ionization stages cannot be provided within our trap set-up. It is limited by the maximum voltage of about 10 kV which can safely be applied to the electrodes. The future scenario is set by the HITRAP project at GSI [26]: at energies of about 400 MeV per nucleon every desired charge state of high-*Z* elements is produced by stripping electrons in a thin foil. They are injected into the experimental storage ring (ESR), electron cooled and decelerated to an energy of 4 MeV/nucleon. After ejection from the ring they are further decelerated by radio-frequency quadrupole structures and finally injected into a Penning trap. After cooling to 4 K they are transported to a similar trap as used for our previous experiments where *g* factor measurements will take place. The system is presently under construction at GSI.

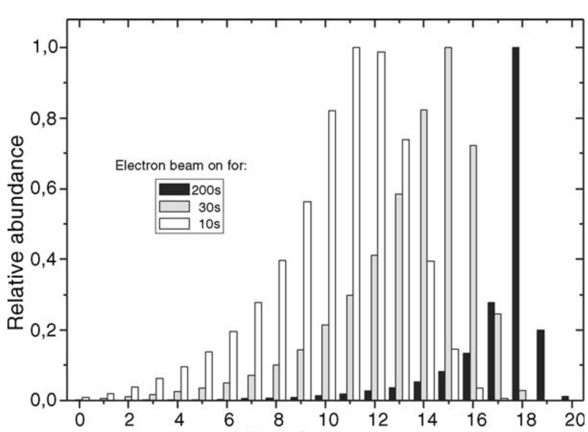

Fig. 8. Simulation of highly charged Ca ion production in an EBIT-like trap showing that the desired hydrogen-like ions can be obtained within a few minutes of charge breeding using an electron energy of 7 keV and a current density of 0.5 A/cm^2.

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